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LETTER TO THE EDITOR

The critical behaviour of systems with correlated defects

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Abstract. For the system in which correlations between quenched defect positions fall as $|x-x'|^{-a}(a=4-\varepsilon-\varepsilon_d>0)$ the ε, ε_d-expansions of the critical exponents are obtained up to the third order. It is shown that if the task of finding the critical exponents for a system with uncorrelated quenched point defects is solved at order ε^m of the ε-expansion, then the critical exponents for the system considered can be simply obtained up to $(m+1)$th order of the ε, ε_d expansion. The calculations are carried out completely in the special case of the uniaxial magnet.

The space correlated quenched defects lead to a critical behaviour type other than the usual one for models with the standard quenched point defects (Khmelnitskii 1975, Lubensky 1975). The case of the systems with correlations between the point quenched impurities falling by a power law $|x-x'|^{-a}(a\equiv 4-\varepsilon-\varepsilon_d>0)$ at large distances (or $\sim u+wk^{-\varepsilon_d}\equiv c(k)$ after the Fourier transformation) was considered by Dorogovtsev (1981) and Weinrib and Halperin (1983). The critical exponents are $\nu=2/(4-\varepsilon-\varepsilon_d)$ and $\alpha=-2\varepsilon_d/(4-\varepsilon-\varepsilon_d)$ exactly when the appropriate fixed point with $w^*\neq 0$ is stable. Here $\varepsilon=4-D$ where D is the space dimension. The other exponents were calculated only to order ε, ε_d. It will be shown in this letter that using information about the critical behaviour of a system with the uncorrelated defects, one can easily obtain the critical exponents of the model under consideration.

It is convenient for us to employ the Ginsburg (1975) approach for the derivation of the Gell-Mann–Low equations although one can work in any other technique. The Gell-Mann–Low equations for the model of the $g\varphi^4$ type with quenched point defects are of the form

$$\begin{aligned} g' &= g(\tfrac{1}{2}(\varepsilon-2\eta(g,u))-f_g(g,u))\equiv\tfrac{1}{2}\varepsilon g-\tilde{f}_g(g,u),\\ u' &= u(\tfrac{1}{2}(\varepsilon-2\eta(g,u))-f_u(g,u))\equiv\tfrac{1}{2}\varepsilon u-\tilde{f}_u(g,u), \end{aligned}\tag{1}$$

where $(\ )'\equiv\partial/\partial\ln\varkappa^{-2}$, $\varkappa$ is the inverse correlation length. The correlation length exponents ν and η are

$$\begin{aligned} \nu^{-1} &= 2-\eta(g,u)-2t(g,u)\equiv 2-2T(g,u),\\ \eta &= \eta(g,u), \end{aligned}\tag{2}$$

where $t(g,u)$ is the one irreducible three-point vertex. As usual, the ladder type of diagrams do not contribute to $gf_g(g,u)$ and $uf_u(g,u)$.

There are three sorts of vertices used in the diagram technique: simple four-point vertices g, dashed and wavy lines, corresponding respectively to contributions u and

$wk^{-\varepsilon_d}$ to the correlator $c(k)$. The principles of the diagrams selection on the right-hand side of the equations (3) are given in Dorogovtsev (1981) and Weinrib and Halperin (1983). The Gell-Mann–Low equations for the system considered are of the form

$$\begin{aligned} g' &= g(\tfrac{1}{2}(\varepsilon - 2\eta(g, u, w)) - f_g(g, u, w)), \\ u' &= u\tfrac{1}{2}(\varepsilon - 2\eta(g, u, w)) - f_u(g, u, w), \\ w' &= w(\tfrac{1}{2}(\varepsilon + \varepsilon d - 2\eta(g, u, w)) - f_w(g, u, w)). \end{aligned} \tag{3}$$

It can be seen that the correlation length exponents of the system with the correlated defects are given by

$$\nu^{-1} = \tfrac{1}{2}(4 - \varepsilon - \varepsilon_d) = 2 - \eta(g, u, w) - t(g, u, w) = 2 - 2T(g, u + w) - \varepsilon_d w + \dots \tag{4}$$

$$= \nu^{-1}(g, u + w) - \varepsilon_d w + \dots, \tag{5}$$

$$\eta = \eta(g, u, w) = \eta(g, u, w) + \varepsilon_d w + \dots.$$

(The contribution $\varepsilon_d w$ is from the one-loop diagram.) Then

$$f_g(g, u, w) = f_g(g, u + w) \tag{6}$$

and

$$f_u(g, u, w) + w f_w(g, u, w) = (u + w) f_u(g, u + w). \tag{7}$$

Only the diagrams, obtained from the all allowed one irreducible four-point vertices by selection the ones with one pair of external lines entering into the wavy vertex w, contribute to the $wf_w(g, u, w)$. Therefore

$$f_w(g, u, w) = 2t(g, u, w), \tag{8}$$

where $t(g, u, w)$ is the usual three-point vertex introduced in (4). Using the relations obtained, we find the Gell-Man–Low functions for a system with the correlated defects:

$$\begin{aligned} g' &= \tfrac{1}{2}\varepsilon g - \varepsilon_d g w - \tilde{f}_g(g, u + w) + \dots, \\ u' &= \tfrac{1}{2}\varepsilon u - \varepsilon_d u w - \tilde{f}_u(g, u + w) - 2wT(g, u + w) + \dots, \\ w' &= \tfrac{1}{2}(\varepsilon + \varepsilon_d) w - \varepsilon_d w^2 - 2wT(g, u + w) + \dots. \end{aligned} \tag{9}$$

Note that the terms with $\varepsilon_d w$ allowing calculation of critical exponents up to the fourth order of the ε, ε_d expansion, are taken into account in (9).

The equations (1) for the one-component magnet with quenched point defects are given by (Shalaev 1977)

$$\begin{aligned} g' &= \tfrac{1}{2}\varepsilon g - \tfrac{3}{2}g^2 + 6gu + 41gu^2 - 23g^2u + \tfrac{17}{6}g^3 + \dots, \\ u' &= \tfrac{1}{2}\varepsilon u - gu + 4u^2 + \tfrac{5}{6}g^2u - 11gu^2 + 21u^3 + \dots. \end{aligned} \tag{10}$$

The exponents are

$$\eta = \tfrac{1}{6}g^2 - gu + u^2 + \dots, \tag{11}$$

$$\nu^{-1} = 2(1 - \tfrac{1}{2}g + u + \tfrac{5}{12}g^2 - \tfrac{5}{2}gu + \tfrac{5}{2}u^2 + \dots). \tag{12}$$

Using (10)–(12) and (2), (5), (9), we obtain

$$\eta = \varepsilon_d w + \tfrac{1}{6}g^2 - g(u + w) + (u + w)^2 + \dots \tag{13}$$

and

$$
\begin{aligned}
g' = {} & \tfrac{1}{2}\varepsilon g - \tfrac{3}{2}g^2 + 6gu + 6gw + \tfrac{17}{6}g^3 - 23g^2u - 23g^2w \\
& + 41gu^2 + 82guw + 41gw^2 - \varepsilon_d gw, \\
u' = {} & \tfrac{1}{2}\varepsilon u - gu + 4u^2 + 6uw + 2w^2 + \tfrac{5}{6}g^2u - 17guw - 11gu^2 \\
& - 6gw^2 + 21u^3 + 58u^2w + 53uw^2 + 16w^3 - \varepsilon_d uw, \\
w' = {} & \tfrac{1}{2}(\varepsilon + \varepsilon_d)w - gw + 2uw + 2w^2 + \tfrac{5}{6}g^2w \\
& - 5guw - 5gw^2 + 5u^2w + 10uw^2 + 5w^3 - \varepsilon_d w^2.
\end{aligned}
\tag{14}
$$

(Really there is no need to take into account the last terms with $\varepsilon_d w$ at the third-order calculations.) For simplicity we consider now the special case of the uniaxial magnet ($\varepsilon = 0$). The coordinates of the appropriate fixed point of equations (14) are

$$
g = \varepsilon_d + \tfrac{1}{36}\varepsilon_d^2, \qquad u = \tfrac{1}{4}\varepsilon_d - \tfrac{17}{144}\varepsilon_d^2, \qquad w = \tfrac{53}{288}\varepsilon_d^2. \tag{15}
$$

Then we find the critical exponents

$$
\begin{aligned}
\eta &= -\tfrac{1}{48}\varepsilon_d^2 + \tfrac{265}{1728}\varepsilon_d^3 + \ldots, \\
\gamma &= 1 + \tfrac{1}{4}\varepsilon_d + \tfrac{7}{96}\varepsilon_d^2 - \tfrac{101}{1728}\varepsilon_d^3 + \ldots.
\end{aligned}
\tag{16}
$$

The critical exponents only for $n = 1$ and $\varepsilon = 0$ were obtained above. However, it is clear that the calculations pursued could be easily generalised to the case of other values n and ε. So if the Gell-Mann–Low equations for the magnet with non-correlated quenched defects are known, then one can also find critical exponents for the system considered. Actually, if for the system with the standard defects the exponent γ is known to order ε^m, then for the correlated defects case one can calculate it up to the $(m+1)$th order of the ε, ε_d expansion.

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